Probabilistic Operator Algebra Seminar

Organizer: Dan-Virgil Voiculescu

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Title: Stochastic optimal control in the free probability setting

Motivated by parallels between mean field games and free probability we develop a theory of stochastic control problems and Hamilton-Jacobi equations for real-valued functions of variables in a tracial von Neumann algebra. Note that the conjectured large deviations rate function for free Brownian motion from Biane, Capitaine and Guionnet's work is an example of such a stochastic control problem. Our models include a free Brownian motion and also a classical Brownian motion motivated by the common noise in mean field game theory. One of the key technical points is that the infimum of some function depends on the choice of ambient von Neumann algebra and filtration and free Brownian motion, and therefore, we consider varying the choice of ambient algebra as well, in the same way as the ambient algebra is allowed to vary in Biane and Voiculescu's free Wasserstein distance. To get a well-defined infimum we thus rely on a pasting lemma that allows us to amalgamate two given free filtrations and Brownian motions. Under certain convexity assumptions on the cost functions, we can show that the non-commutative value function agrees with the large-n limit of the analogous value functions on the space of matrices. This is based on joint work with Wilfrid Gangbo, Kyeongsik Nam and Aaron Palmer.