Probabilistic Operator Algebra Seminar

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Title: *Free probability and spin-glass theory*

We use free probability and random matrix theory to analyze a certain spin glass model. Spin glasses are a family of statistical mechanical models where sites of a graph are assigned spins (often plus/minus 1) and generate a random Hamiltonian operator. In the case of a complete graph and Gaussian interactions between neighbors, known as the Sherrington-Kirkpatrick model, finding the lowest eigenspace (ground state) of the Hamiltonian amounts to maximizing the quadratic form $\langle x, Ax \rangle$ over $x \in \{-1, 1\}^n$, where A is a GOE random matrix. Our goal is to find a near-maximizer in polynomial time in n, the size of the matrix. Adapting the Hessian ascent method Subag used for the sphere, we construct an objective function of the form $u(t, x) = \langle x, Ax \rangle - \sum_{j=1}^n \Lambda(t, x) - \int_t^1 sF_\mu(s) ds$. Here Λ and F_μ are functions related to the Parisi PDE which describes the large n solution. The function u has the property that it agrees at the corners of the cube with the quadratic form we want to maximize , but in the interior there is a path from 0 to the corner along which the maximum is achieved. We describe a randomized algorithm that approximately follows this path, at each step using the Hessian of the objective function to determine which direction to go with higher probability. We construct a matrix that approximately projects into the top eigenspace of the Hessian using the imaginary part of a resolvent. Using free subordination theory and concentration of measure, we control the diagonal entries of this matrix, which in turn is used to analyze the empirical distribution of the coordinates of the vectors obtained in the algorithm, and show convergence to an SDE associated to the Parisi equation in the large n limit.