Probabilistic Operator Algebra Seminar

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Title: *Self-normalized Sums in Free Probability Theory*

Given a sequence $(X_i)_{i\in\mathbb{N}}$ of free self-adjoint random variables, we study the corresponding self-normalized sum U_n defined by

$$U_n := V_n^{-\frac{1}{2}} S_n V_n^{-\frac{1}{2}}, \quad S_n := \sum_{i=1}^{n} X_i, \quad V_n^2 := \sum_{i=1}^{n} X_i^2.$$

We prove that the distribution of U_n converges weakly to Wigner's semicircle law under appropriate conditions and estimate the rate of convergence with respect to the Kolmogorov distance. In the case of free identically distributed self-adjoint bounded random variables, we retrieve the standard rate of order $n^{-\frac{1}{2}}$ up to a logarithmic factor, whereas we obtain a rate of order $n^{-\frac{1}{4}}$ in the corresponding unbounded setting. These results can be interpreted as free versions of certain self-normalized limit theorems in classical probability theory. This talk is based on arXiv:2406.13601.