Probabilistic Operator Algebra Seminar

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Title: Signal communication and modular theory.

We propose a conceptual frame to interpret the prolate differential operator, which appears in Communication Theory, as an entropy operator; indeed we write its expectation values as a sum of terms, each subject to an entropy reading by an embedding suggested by Quantum Field Theory. This adds meaning to the classical work by Slepian et al. on the problem of simultaneously concentrating a function and its Fourier transform, in particular to the "lucky accident" that the truncated Fourier transform commutes with the prolate operator. The key is the notion of entropy of a vector of a complex Hilbert space with respect to a real linear subspace, recently introduced by the author by means of the Tomita-Takesaki modular theory of von Neumann algebras. We consider a generalization of the prolate operator to the higher dimensional case and show that it admits a natural extension commuting with the truncated Fourier transform; this partly generalizes the one-dimensional result by Connes to the effect that there exists a natural selfadjoint extension to the full line commuting with the truncated Fourier transform.