

# Probabilistic Operator Algebra Seminar

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Title: *Free Integral Calculus*

We study the problem of conditional expectations in free random variables and provide closed formulas for the conditional expectation of resolvents of arbitrary non-commutative polynomials in free random variables onto the subalgebra of an arbitrary subset of the variables. More precisely, given a linearization of the resolvent we compute a linearization of its conditional expectation. The coefficients of the expressions obtained in this process involve certain Boolean cumulant functionals which can be computed by solving a system of equations. On the way towards the main result we introduce a non-commutative differential calculus which allows to evaluate conditional expectations and the said Boolean cumulant functionals. Reference: <https://arxiv.org/abs/2311.04039>