Probabilistic Operator Algebra Seminar

Organizer: Dan-Virgil Voiculescu

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Title: *An Application of Free Probability in the Study of Noncommutative Constraint Satisfaction Problems*

In this talk I explore an application of free probability in our recent work on operator – or noncommutative – variants of constraint satisfaction problems (CSPs). These higher-dimensional variants are a core topic of investigation in quantum information, where they arise as nonlocal games and entangled multiprover interactive proof systems (MIP*). The idea of higher-dimensional relaxations of CSPs is also important in the classical computer science literature. For example, since the celebrated work of Goemans and Williamson on the Max-Cut CSPs, higher dimensional vector relaxations have been central in the design of approximation algorithms for classical CSPs. We introduce a framework for designing approximation algorithms for noncommutative CSPs. In the study of classical CSPs, k-ary decision variables are often represented by k-th roots of unity, which generalize to the noncommutative setting as order-k unitary operators. In our framework, using representation theory, we develop a way of constructing unitary solutions from SDP relaxations, extending the pioneering work of Tsirelson on XOR games. Then we introduce a novel rounding scheme to transform these unitary solutions to order-k unitaries. Our main technical innovation here is a theorem guaranteeing that, for any set of unitary operators, there exists a set of order-k unitaries that closely mimics it. As an integral part of the rounding scheme, we prove a random matrix theory result that characterizes the distribution of the relative angles between eigenvalues of random unitaries using tools from free probability. Based on joint work with Eric Culf and Taro Sprig arXiv:2312.16765.